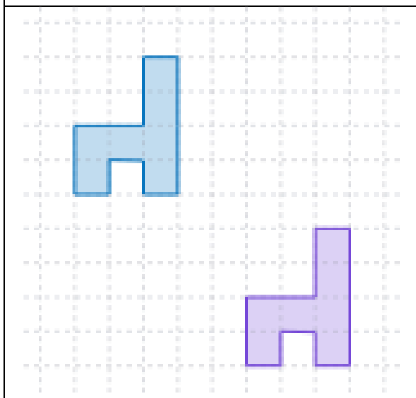
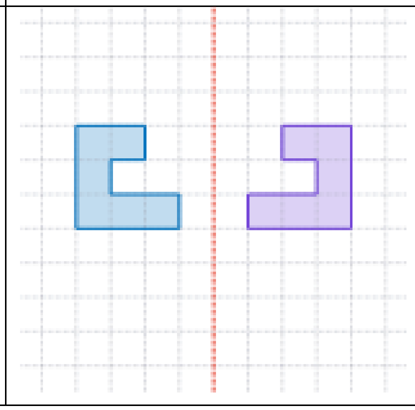
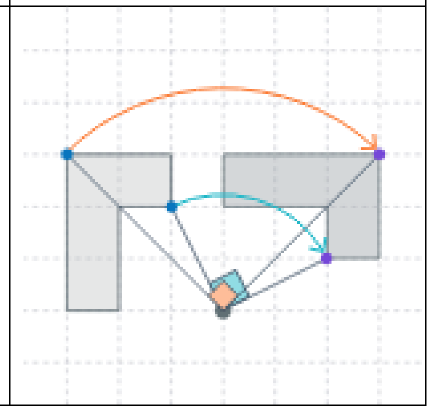
Transformations and Congruence:

What is transformation?

There are 3 different types of transformations.

Translations: Movement of a shape by a certain number of units on the x-y plane.	**Reflections: F**lip a shape or a line across a line.	**Rotations:** Move an object from a certain angle.
		

Keep in mind that the properties of the shape remain the same in ways such as but not limited to;

1. Area stays the same.
2. Side length stays the same.
3. All internal angles remain the same.

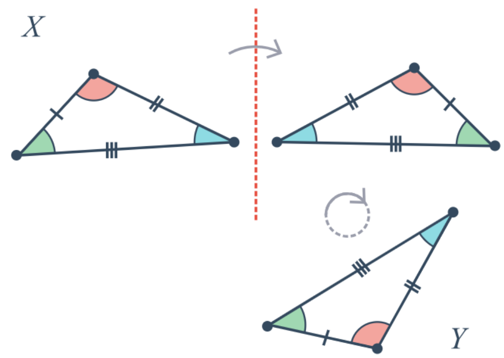
Rigid Transformations:

- Translations slide shapes around. Reflections flip shapes across a line. Rotations rotate shapes around a point. These rigid transformations preserve the area, side lengths, and internal angles of the shape.
- Reflections, translations and rotations can be thought of as happening to the individual points of a shape.
- If a shape is reflected but remains unchanged, then that line is an axis of symmetry.

Congruence:

We describe two shapes as congruent if we can perform steps of transformations that allow us to make one shape into the other.

Two shapes are congruent if corresponding side lengths and angles are the same.



A bisector is a line that cuts something into two equal halves. An angle bisector divides an angle into two angles of equal size, and a side bisector divides a side into two segments of equal length.

In an isosceles triangle, the line through a vertex bisecting the opposite side is also an angle bisector. This line is an axis of symmetry for the triangle and meets the base at right angles.

Proving congruence:

There are 4 main proofs for congruence

1. SSS: Three pairs of equal sides
2. SAS: Two pairs of equal sides with an equal included angle
3. AAS: Two pairs of equal angles and one pair of equal sides
4. RHS: Both have right angles, equal hypotenuses, and another equal side

If two triangles are congruent, then:

- The sides in the same relative position are equal and are called corresponding sides (of congruent triangles).
- The angles in the same relative position are equal and are called corresponding angles (of congruent triangles).

We can use triangle congruence to understand the values of missing sides and angles in congruent triangles.

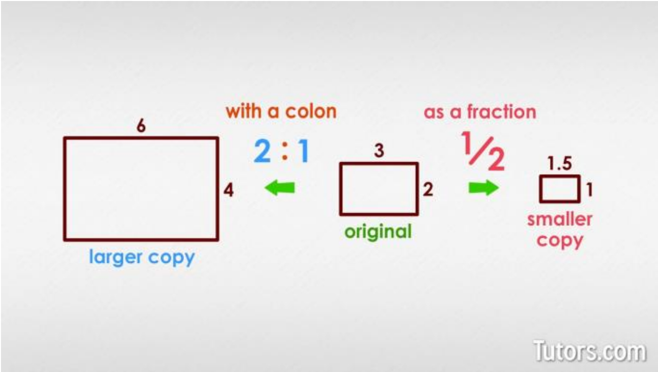
What to look out for when proving congruence?

1. Parallel lines. (Alternate or Corresponding angles)
2. Markings on side lengths.
3. Vertically opposite angles around a point.
4. Properties of quadrilaterals.
5. Pairs of sides in a common ratio.
6. Sides or angles that are common in both triangles.

Similarity in Triangles:

Scale factor:

Scale factor is a number that enlarges or diminishes a shape to form a new shape with different side lengths and same angles.



Two triangles are considered to be similar if one of them can be scaled up or down in size, and then rotated and/or reflected to match the other.

Sides in fixed ratio

We can think of similarity as a weaker version of congruency, where corresponding distances do not need to be equal but instead are in some fixed ratio.

The fixed ratio of distances between two similar figures is called the scale factor.

Similarity tests for triangles

There are four tests that we can use to test similarity between two triangles. If any one of these tests is satisfied, then the two triangles must be similar. These four tests are:

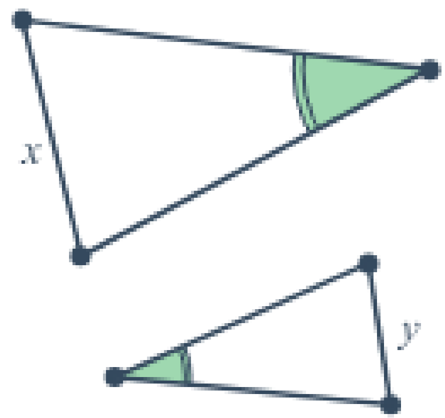
- AAA: Three pairs of equal angles
- SSS: Three pairs of sides in the same ratio
- SAS: Two pairs of sides in the same ratio and an equal included angle
- RHS: Both have right angles, and the hypotenuses and another pair of sides are in the same ratio

These similarity tests can be proved to work in the same way that the congruency tests work, except with sides in fixed ratio rather than needing to be equal.

It is important to understand that all triangles' internal angles have a sum of 180˚, therefore, having two matching pairs of angles, means the third will also be the same, proving AAA.

To determine which sides are corresponding in two potentially similar triangles, we can use their positions with respect to any matching angles. If there is a common sized angle in both triangles, then the sides opposite those angles will correspond.

In this case, the sides labelled x and y are corresponding since they are opposite the same sized angle.



Similarly, if side lengths are given and two pairs of sides are in a fixed ratio between the two triangles, the angles between these pairs of sides will correspond.

Area and Volume Scale Factor:

All pairs of corresponding sides in similar figures are in the same ratio, and we can use this ratio to find the scale factor.

Since this scale factor affects the length of sides, it is also called the length scale factor or linear scale factor.

But what happens to the area of a figure when we enlarge it by a length scale factor? Does it also enlarge?

Suppose we have a square with side lengths 2 cm, and we enlarge it by a length scale factor of 3.

The sides have increased by a scale factor of 3, however, the area has increased by a scale factor of 9.

The same rules apply for volume, however, instead of squaring the side scale factor, we cube the side scale factor.

We can use this in tests to find the length of unknown sides by understanding the relationship between the side and area scale factor.

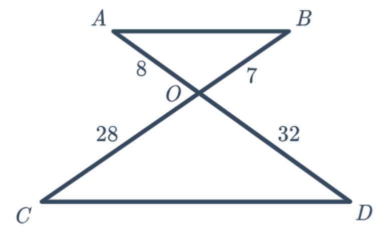
Further Triangle Proofs

After proving that two triangles are similar or congruent, we gain access to all the properties of similar or congruent triangles.

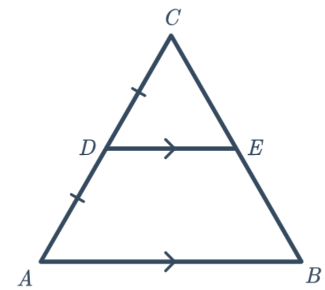
If we can prove that two triangles are congruent using any of the congruence tests, we have also proved that and angle or side in one triangle must be equal to the corresponding angle or side in the other.

The same applies if we can prove two triangles are similar, except instead of equal sides we get sides in a common ratio.

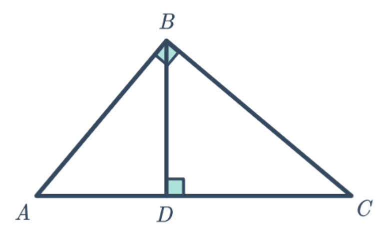
9 Prove that AB is parallel to CD for the following diagram:



10 Prove that $CE = EB$ for the following diagram:



11 Prove that $BD^2 = AD \times DC$ for the following diagram:



How to use correct notation:

1. When speaking about a side, mention the two points it is subtended by. For example, in question ten, the parallel lines can be referred to as DE or AB, or the other way around.
2. Similarly for angles, the letter in the middle refers to the point at which the angle is made, take question 11 as an example, where the right-angle forms at the base of the triangle. This angle can be referred to as BDC or CDB.

Solution to question 10:

$$2CD = CA$$

$$\angle CDE = \angle CAB$$

$$Because\ DE \parallel AB$$

$$\angle BCA = \angle ECD$$

$$\angle CED = \angle CBA$$

$$\therefore \Delta CDE \cong \Delta ABC$$

$$\frac{2CD}{CD} = \frac{CB}{CE}$$

$$\frac{CB}{CE} = 2$$

$$CE = EB$$